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SPECTRAL INFLUENCE FUNCTIONS OF BOUNDARY EFFECTS IN PROBLEMS DEALING WITH CONTROL OF THERMAL OBJECTS

Yu. M. Matsevityi, A. P. Slesarenko, and O. S. Tsakanyan

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We describe a method for solving problems dealing with the optimum control of a thermal object that is based on high-speed action where limitations are imposed on the control function and on the thermal state.

The problem of optimum high-speed control of a thermal object whose dynamics are described by the following equation of heat conduction

$$\frac{\partial T(x,\ y,\ \mathrm{Fo})}{\partial\,\mathrm{Fo}} = \nabla^2 T(x,\ y,\ \mathrm{Fo}), \tag{1}$$

consists in the determination of such control boundary effects, expressed by piecewise polynomial functions

$$q_i(x,\ y,\ \mathrm{Fo}) = \sum_{j=0}^{m_i} a_{ij}(\mathrm{Fo})\,\xi^j;\ i = 1,\ 2,\ \ldots,\ n;\ j = 0,\ 1,\ \ldots,\ m_i, \tag{2}$$

which would ensure the transition of the object from a given initial thermal state T(x, y, 0) to the final state T(x, y, Fo_k) within a minimum of time, with satisfaction of the limitations imposed both on the controling action (the external limitation)

$$q_{\min} \leqslant q_i(x,\ y,\ \mathrm{Fo}) \leqslant q_{\max}, \tag{3}$$

as well as on the thermal state of the object (an internal system of limitations)

$$T(x,\ y,\ \mathrm{Fo}) \leqslant T_{\mathrm{per}}, \tag{4}$$

$$\Delta T(x,\ y,\ \mathrm{Fo}) \leqslant \Delta T_{\mathrm{per}}. \tag{5}$$

We will adopt the attainment of the maximum possible rate of temperature change in the object in conjunction with the given limitations (3)-(5) as the criteria of optimum control.

Applying an implicit finite-difference approximation to Eq. (1), for k-th instant of time we obtain

$$\nabla^2 T^{(k)}(x,\ y) - (\Delta\mathrm{Fo})^{-1} T^{(k)}(x,\ y) = -(\Delta\,\mathrm{Fo})^{-1} T^{(k-1)}(x,\ y). \tag{6}$$

If we specify the spectral component ξ^j as the controling action on the i-th segment of the object's boundary, with zero actions specified for the remaining (n − 1) segments, and if we solve system of equations (6) with the zero initial conditions T(x, y, 0) = 0, we will obtain the spectral influence function (SIF) $W_{ij}(x, y)$ [1].

Having thus determined the remaining SIF, we will represent the temperature at the observation point s for the k-th instant of time in the form

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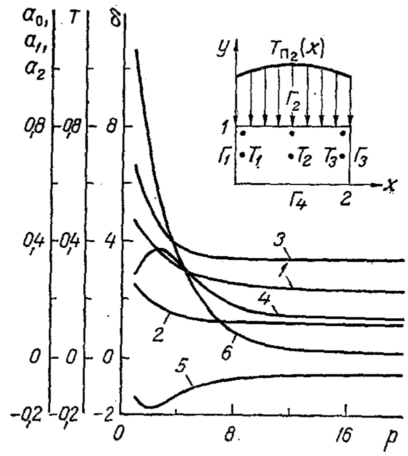

Fig. 1. Change in the dimensionless temperature at the points of observation: 1) T_1; 2) T_2; change in the control parameters: 3) a_0 ; 4) a_1 ; 5) a_2) and $\delta = \Delta T - \Delta T_{per}$; 6) δ, as a function of the iteration number. T and are dimensionless quantities; δ, %.

$$T^{(k)}(x_s,\ y_s) = \sum_{i=1}^{n}\sum_{j=0}^{m_i} a_{ij}^{(k)} W_{ij}(x_s,\ g_s) + T_H^{(k)}(x_s,\ y_s). \tag{7}$$

The first term in the right-hand side of Eq. (7) characterizes the object's reaction to the controling action, while the second term $T_H^{(k)}(x_s, y_s)$ represents the reaction of the object to the initial action [the temperature $T^{(k-1)}(x, y)$].

The rate at which temperature changes at each time interval is represented in the form of the sum of the following two components

$$[T^{(k)}(x_s,\ y_s) - T^{(k-1)}(x_s,\ y_s)](\Delta\mathrm{Fo})^{-1} = \tag{8}$$
$$= \left[\sum_{i=1}^{n}\sum_{j=0}^{m_i} a_{ij}^{(k)} W_{ij}(x_s,\ \ y_s) + T_H^{(k)}(x_s, y_s) - T^{(k-1)}\ (x_s,\ y_s)\ \right](\Delta\mathrm{Fo})^{-1},$$

from which the first is associated with the flow of heat to the surface of the object as a control function at k-th instant of time, while the second component is related to the temperature derived as a result of controlling the thermal regime of the object at the previous instant of time.

In order to solve the control problem, it is necessary at each time interval:

1. to determine SIF of the boundary actions

$$W_{ij}(x,\ y);\ i = 1,\ 2,\ \dots,\ n;\ j = 0,\ 1,\ \dots,\ m_i.$$

2. to calculate the maximum possible increments $\Delta T_{max}(x, y)$ in temperature using Eq. (6), within the limits of the time interval ΔFo for specified maximum permissible boundary actions $q_{max}(\xi)$ and zero initial conditions.

3. to establish the maximum temperature increments at the observation points exhibiting coordinates (x_s, y_s) and (x_L, y_L), the difference between the these being specified by limitation (5).

4. to calculate the function $T_H{}^k(x, y)$, solving Eq. (6) with the boundary actions equal to zero, and the initial conditions $T^{(k-1)}(x, y)$.

5. to verify satisfaction of limitations (4) and (5) for the k-th interval at the observation points

$$\Delta T_{\max}(x_s,\ y_s) + T_H^{(k)}(x_s,\ y_s) \leqslant T_{\mathrm{per}}, \tag{9}$$

$$[\Delta T_{\max}(x_s,\ y_s) + T_H^{(k)}(x_s,\ y_s)] - [\Delta T_{\max}(x_L, y_L) + T_H^{(k)}(x_L,\ y_L)] \leqslant \Delta T_{\mathrm{per}}. \tag{10}$$

6. satisfaction of conditions (9) and (10) indicates that the rate of change in temperature for the k-th instant of time is maximum, since it is determined from the limitation (3) imposed on the control action.

7. if conditions (9) and (10) are not satisfied, the problem is solved by an iteration method. The rate of change in temperature diminishes as a consequence of the reduced

intensity of the control actions whose magnitude is determined through solution of the inverse heat-conduction problem in which T_{per} is taken as the "observed" temperature, so that

$$\Delta T_{(p)}^{(k)}(x_s, y_s) = T_{per} - T_H^{(k)}(x_s, y_s), \tag{11}$$

while

$$\Delta T_{(p)}^{(k)}(x_s, y_s) = \Delta T_{per} + \Delta T_{(p-1)}^{(k)}(x_L, y_L) + T_H^{(k)}(x_L, y_L) - T_H^{(k)}(x_s, y_s), \tag{12}$$

where p is the number of the iteration.

8. the parameters of the boundary actions $a_{ij}^{(k)}$ are determined from the solution of the following system of linear algebraic equations (SLAE)

$$\sum_{i=1}^{n} \sum_{j=0}^{m_i} a_{ij}^{(k)} W_{ij}(x_s, y_s) = \Delta T_{(p)}^{(k)}(x_s, y_s), \tag{13}$$

s = 1, 2, ..., N, where N represents the number of observation points.

9. the temperature increments are calculated at points with coordinates (x_L, y_L) from the expression

$$\Delta T_{(p)}^{(k)}(x_L, y_L) = \sum_{i=1}^{n} \sum_{j=0}^{m_i} a_{ij} W_{ij}(x_L, y_L) \tag{14}$$

and condition (10) is verified

$$[\Delta T_{(p)}^{(k)}(x_s, y_s) + T_H^{(k)}(x_s, y_s)] - [\Delta T_{(p)}^{(k)}(x_L, y_L) + T_H^{(k)}(x_L, y_L)] \leqslant \Delta T_{per}.$$

10. if condition (10) is not fulfilled, the temperature increment $\Delta T_{p+1}{}^{(k)}(x_s, y_s)$ is predicted on the basis of expressions (11) and (12), and operations 8, 9, and 5 are carried out. The iteration process will be curtailed on satisfaction of conditions (9) and (10), with the specified accuracy, subsequent to which we turn to the determination of the parameters at the next time interval (k + 1).

Evidence as to the conclusion of the solution for the problem will be attained on reaching the points at which the specified temperature can be observed (given allowable divergences), such as will characterize the final thermal state of the object.

If system (13) has been overdetermined N > mn, and the method of least squares is used to bring the system of linear algebraic equations (13) to normal form.

In the algorithm for the solution of the control problem which we are examining here, at each time interval the initial approximations of the temperature increments at the observation points are set as high as possible, and this corresponds to the maximum rate of change in temperature, since ΔT_{max} is a reaction to the maximum permissible control action q_{max}. Consequently, the object is extended beyond the regional boundary of the external limitation (3). The iteration process is ascribed to the fact that with a maximum rate of change in temperature the internal system of limitation (4) and (5) may not be fulfilled. Satisfaction of this system of limitations, given the maximum rate of change in temperature, will indicate that the final coordinates of the object will slide along the regional boundary of the internal limitations (4) and (5) [2].

Application of the spectral influence functions for the boundary actions in the solution of the control problem makes it possible to change from a multiple solution of direct problems, such as is necessary for verification of the fulfillment of the system of limitations, to a one-time solution of the inverse heat-conduction problem in which the temperature values found at the regional boundary of the internal limitations is used as the "measured" temperature value, and as a result of the solution of this problem the sort control action is found. In addition to the reduction in the order of the solved system of equations (because of the use of the spectral influence functions), this circumstance enhances a reduction in the time required for the solution of the control problem.

As an example, let us examine the problem of the optimum control in the heating of an infinite rectangular prism whose sides are in a ratio of 1:2.

Figure 1 shows the changes in the dimensionless temperatures at the points $T_1(0.1; 0.5)$ and $T_2(1; 0.5)$, as well as in the parameters of the control actions a_0, a_1, a_2, and $\delta = \Delta T - \Delta T_{per}$, dependent on the iteration number.

At the boundary segments Γ_1, Γ_2, Γ_3 the surface temperature $T_s = 0$ represents the initial condition $T(x, y, 0) = 0$.

At the boundary segment Γ_2 the surface temperature $T_s = a_0 + a_1 x + a_2 x^2$, $0 \le x \le 2$ is the sort function under the following limitations:

$$T_{s_2}(x, 1) \leqslant 1,$$

$$T(x_i; 0.9) - T(x_i; 0.5) \leqslant 0{,}1; \ i = 1, 2, 3.$$

The solution of the problem was found with $\Delta x = \Delta y = 0$, $\Delta Fo = 0.1$. The condition

$$\delta = \max |T(x_i; 0.9) - T(x_i; 0.5) - 0.1| \leqslant 10^{-3}$$

was fulfilled in the 20-th iteration ($p = 20$). Here the parameters of the control action were equal to: $a_0 = 0.345$; $a_1 = 0.1424$; $a_2 = 0.007114$.

The data in these results prove the effectiveness of the proposed method, which may be used to solve two-dimensional heating control problems (or cooling) in solids.

NOTATION

T, temperature; x, y, spatial coordinates; Fo, dimensionless time; q, heat flow (control function); ξ, boundary contour coordinate; a_{ij}, boundary-action function parameters; W_{ij}, spectral influence functions of boundary actions; δ, magnitude of divergence between modeled function and specified limitation. Subscripts: s and L, observation points; k, number of time intervals; p, iteration number; Γ, boundary segment; s, surface; per, permissible magnitude.

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